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Abstract

How does cost uncertainty affect the welfare consequences of an oligopoly? To answer this question, we investigate a Cournot oligopoly in which firms produce a homogeneous commodity and market entry is feasible. Marginal costs are unknown ex-ante, i.e. prior to entering the market. They become public knowledge before output choices are made. We show that uncertainty induces additional entry in market equilibrium and also raises the socially optimal number of firms. Since the first change dominates, the excessive entry distortion is aggravated. This prediction is robust to various extensions of the analytical set-up. Furthermore, the welfare loss due to oligopoly tends to increase with uncertainty.

Keywords: Oligopoly, Excessive Entry, Uncertainty, Welfare

JEL Classification: D 43, L 13

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1 Introduction

An increase in the number of competitors in an oligopolistic market generally reduces each firm's output. It is a well-established prediction that in the presence of such business-stealing effect, there will be excessive entry in a homogeneous Cournot oligopoly with economies of scale, relative to the number of firms a social planner would choose. The reason is that each entrant ignores the adverse repercussion of its decision on other firms' production levels and profits. While a different distribution of profits has no direct welfare consequences, the profit effect raises the potential entrant's incentives to take up production. Moreover, it dominates the welfare-enhancing impact of entry on consumer surplus, which firms do not take completely into account either.

Most pertinent analyses of the excessive entry prediction consider a world of certainty. Therefore, firms can fully anticipate the decisions of other agents and the ensuing payoffs when deciding about entry. In this paper, we depart from this simplifying assumption and consider the impact of cost uncertainty. Because uncertainty affects the expected level of output and, thereby, expected profits, imperfect knowledge about costs when determining whether to set up production alters the number of firms in market equilibrium. Since the same is true with regard to the number of firms preferred by a social planner, a priori the impact of cost uncertainty on excessive entry, i.e., on the difference between the social planner's choice and the market outcome, is ambiguous.

In our analytical set-up, firms face uncertainty with regard to productivity or, which is analytically equivalent, marginal costs. While the ex-post distribution is known ex-ante, the actual level of costs, which each firm faces, is revealed only after entry has taken place, but before production decisions are made. Hence, entry has to be decided upon in a world of uncertainty, whereas firms choose production levels having full knowledge of their own and other firms' costs. This informational structure captures the feature that uncertainty is greatest prior to market entry and diminishes subsequently.[1]

To illustrate our setting, assume that the costs of an input are variable, but can be established on average before entry is decided upon. An example may be a setting in

[1]Following the Knightean distinction between risk and uncertainty, we consider a situation with risky outcomes, which is often referred to as a set-up with price or cost variability. Since risk attitudes play no role in our setting, we employ the term cost uncertainty for simplicity.

which the distribution of qualifications of the labour force in a specific country or region is known. However, it may well be that those employees, which a firm hires after entering a particular market, have an above or below average qualification and, thus, productivity. Accordingly, there is uncertainty about productivity and marginal costs prior to entry, which is resolved before output decisions are made.

In our basic framework, uncertainty about productivity does not result in marginal costs in excess of the price. Hence, there is no firm exit. Uncertainty is firm-specific, such that some firms face higher than average costs, while others benefit from lower costs. More specifically, the ex-ante random distribution of marginal costs is also realized ex-post. Therefore, the aggregate outcome can be anticipated prior to entry. Finally, the number of firms is a continuous variable and the cost function is linear.

We find that expected profits increase with cost uncertainty, such that there is more entry in market equilibrium. Furthermore, aggregate output and welfare rise. If the number of firms is chosen in a socially optimal manner, uncertainty has the same qualitative effects. Importantly, uncertainty makes excessive entry more pronounced because the business-stealing externality is aggravated. In addition, we show that the output distortion becomes greater with uncertainty if there are few firms and less severe if the number of firms is sufficiently large. The intuition for this non-linear relationship is that the business-stealing externality is relatively strong if there are few firms. Therefore, the increase in output in market equilibrium is substantial on account of the rise in the number of competitors. The more firms there are, the smaller is the expansion in output in market equilibrium, relative to the socially optimal outcome. Furthermore, if the difference between socially optimal output and the level resulting in market equilibrium declines sufficiently strongly, the welfare loss due to oligopoly initially rises and eventually declines with uncertainty.

In order to ascertain the robustness of our findings, we analyze a number of modifications of the basic set-up. First, we assume a variant of comprehensive instead of firm-specific uncertainty. In particular, all firms face the same marginal costs ex-post. Second, we consider quadratic instead of linear costs. Third, we allow for the exit of firms. All these extensions indicate that the exact modeling of uncertainty does not affect our main results, namely, ex-ante uncertainty aggravates excessive entry. Finally, we restrict the number of firms to be an integer. We additionally use the integer setting to investigate a more general version of comprehensive uncertainty in which the distribution of marginal costs is also random ex-post. For this last modification, we show that there is excessive entry, but that uncertainty does not necessarily aggravate this effect.

In the remainder of the paper, we survey related contributions in Section 2. We outline the model in Section 3 and determine the market equilibrium and the (second-best) optimal outcome in Section 4. Section 5 contains the analysis of uncertainty in the base model. We consider the extensions in Section 6 and provide concluding remarks in Section 7.

2 Literature Review

Our paper is primarily related to three strands of literature. The first analyzes price and cost uncertainty in competitive markets. The second investigates the determinants and welfare effects of excessive entry in oligopoly.[2] The third consists of the few contributions which combine features of both approaches.

The effects of comprehensive price variability, respectively price uncertainty, have initially been investigated by Waugh (1944) and Oi (1961) and looked at in combination in a unified framework by Massell (1969). These authors consider shifts in demand or supply curves in a competitive market and show that consumers or firms can benefit from such (additive) price variability to which firms can respond by adjusting behavior.[3] Massell (1969) further demonstrates that the sum of expected consumer surplus and profits decreases. Turnovsky (1976) clarifies that the detrimental welfare effects carry over to settings in which variability also alters the slope of demand and supply curves. The perti-

[2]Other contributions study the welfare distortions in markets with monopolistic competition. Dixit and Stiglitz (1977) show that the business-stealing effect interacts with a love-of-variety effect, which could lead to insufficient entry. There are many extensions of this basic framework, e.g. by incorporating heterogeneous firms (see Dhingra and Morrow, 2019, Zhelobodko et al., 2012) or by considering heterogeneous sectors (Behrens et al., 2020).

[3]This approach differs from settings in which the realization of the price can only be observed after output decisions have been made and firms are risk-averse (see Baron, 1970, Leland, 1972, Sandmo, 1971). The effects of uncertainty on entry decision have also been looked at in contributions utilizing the real options approach (see, e.g., Dixit, 1989). In this setting, uncertainty alters the value of waiting such that it affects the timing and possibly volume of market entry investments.

nent studies have often been inspired by price variations in agricultural products and the question whether governments should use buffer stocks to reduce price variability over time. In addition, various contributions examine the suitability of expected consumer surplus as an indicator of (consumer) welfare (see, inter alia, Gilbert, 1986, Helms, 1985, Rogerson, 1980, Schlee, 2008, Turnovsky, 1976). Furthermore, Samuelson (1972) argues that the price variation required to achieve the welfare gains derived by Waugh (1944) and Oi (1961) cannot be realized in a general equilibrium setting.

The basic feature of the analyses by Waugh (1944), Oi (1961), and Massell (1969), namely that payoffs are non-linear in prices or unit costs, has been applied in other fields than agricultural economics, such as international trade (Bieri and Schmitz, 1973, Hueth and Schmitz, 1972) or optimal taxation (Goerke, 2011, Hines and Keen, 2018, Kotsogiannis and Serfes, 2014). Moreover, the impact of uncertainty on investment has been looked at in models in which risk-neutral firms first choose the capital stock and subsequently adjust labor input, once an ex-ante uncertain parameter has been revealed (Abel, 1983, Caballero, 1991, Hartman, 1972).[4] In contrast to our setting, these studies do not focus on the consequences of uncertainty for output and welfare.

Turning to the second strand of literature, the excessive entry prediction has been derived by Mankiw and Whinston (1986), Perry (1984), Suzumura and Kiyono (1987), and von Weizsäcker (1980), among others. Follow-up investigations, in which uncertainty does not play a role, confirm the robustness of the prediction.[5] Our paper can also be related to investigations which focus on cost reductions (Chao et al., 2017, Haruna and Goel, 2011, Mukherjee, 2012, Okuno-Fujiwara and Suzumura, 1993) because uncertainty implies that marginal costs fall below the expected value at least for some firms. In these contributions, excessive entry may no longer result and the inefficiencies often depend on the extent of ex-ante cost asymmetries and knowledge spillovers. Both aspects play no role in our analysis. Moreover, we compare changes in the market outcome and the socially

[4] For empirical analyses of the impact of uncertainty on investment, see, for example, Ghosal and Loungani (1996), Henley et al. (2003), Baum et al. (2008), and Fuss and Vermeulen (2008).

[5] For instance, Amir et al. (2014) show that there is excessive entry if the production process is characterized by increasing returns to scale, while Wang (2016) proves the robustness of the prediction in an open economy setting. Moreover, in case of imperfect input markets, the business-stealing effect prevails and leads to excessive entry in many (but not all) considered settings (see, inter alia, de Pinto and Goerke, 2020, Ghosh and Morita, 2007a,b). Polo (2018) and Etro (2014) provide surveys and Mas-Colell et al. (1995) a nice textbook treatment.

optimal situation. The previous analyses have not undertaken such an assessment.

Third, some contributions investigate price and cost variability or uncertainty in Cournot-oligopolies with entry. Creane (2007) assumes that firms incur entry costs but can only start production with an exogenous probability. All incumbents are identical and produce one unit. Imposing an integer constraint, Creane (2007) shows that insufficient entry may occur if the exogenous failure probability is high enough. Silvers (2018) considers a setting in which entry costs can be high or low with equal probability. Firms receive a signal about them and then decide about market entry. If the demand schedule is linear, there is excessive entry: The expected number of excess entrants is minimal if the signal is uninformative. In Deo and Corbett (2009), the production process is characterized by yield uncertainty, that is, the actual output level cannot be inferred from the amount of inputs chosen. Moreover, in contrast to our setting, input quantities are determined before uncertainty is resolved. Deo and Corbett (2009) demonstrate that the number of entrants in market equilibrium first increases and then declines in the extent of uncertainty, and that aggregate expected output is likely to fall, while consumer surplus decreases. Moreover, there is insufficient entry if the variance of output exceeds a critical value.[6] Finally, Hirokawa and Sasaki (2000) analyze a framework in which firms can decide whether to choose output when the choke price is still uncertain or after its realization. Late output decisions are beneficial from a welfare perspective as they can be conditioned on demand fluctuations. Despite this advantageous effect of entry by firms, which decide about output after the uncertain demand parameter has been revealed, entry is generally excessive.

In conclusion, the interaction of entry and output decisions by Cournot oligopolists and of ex-ante uncertainty about marginal costs has not been looked at so far.

3 Model

We consider an oligopolistic market in which n ex-ante identical firms produce a homogeneous good. Firms compete in quantities q and take the choices of competitors as given

[6] Jansen and Özaltın (2018) extend Deo and Corbett's (2009) study by allowing for capacity-constrained firms.

(Cournot-Nash setting). They face entry costs k, $k > 0$, which are sunk, and can arise from an investment which has to be undertaken to establish production facilities. Ex-ante, that is, when deciding about whether to take up production or not, firms only know the probability distribution of marginal production costs c. We model this distribution in a simple manner to clearly establish the various adjustments in firm behavior. In particular, we assume that marginal production costs can either be high, $c_h = c+\epsilon$, or low, $c_l = c-\epsilon$, where $0 \leq \epsilon < \min[c, \sqrt{0.5k}]$ and $c < 1 - 2\sqrt{k - \epsilon^2}$ hold, and the probability of either cost realization is 0.5. The first upper bound for ϵ, namely c, ensures that marginal costs, c_l, of low-cost firms are positive. The second bound, $\sqrt{0.5k}$, guarantees that a high-cost firm produces a positive quantity in equilibrium, as will be demonstrated below. Finally, the restriction on c implicitly imposes an upper bound on k, which ensures that at least one firm finds it profitable to enter the market. The constraint also rules out a setting in which even a low-cost firm has no incentives to produce a positive quantity.

An increase in uncertainty is captured by a mean-preserving spread (cf. Rothschild and Stiglitz, 1970), that is, a rise in ϵ, which is equivalent to an increase in the variance of marginal cost.[7] Once entry decisions have been made, marginal costs become public knowledge. In addition, we assume that the ex-ante distribution of marginal production costs is also realized ex-post.[8] In the terminology of Février and Linnemer (2004), the average impact of the cost shock is zero, while we focus on the heterogeneity effect, that is, uncertainty, which is specific to firms but does not affect the market in aggregate directly.

We do so for numerous reasons: First, we are interested in a firm's decision whether or not to enter a specific market, and this choice is affected by firm-specific uncertainty. Instead, market (or comprehensive) uncertainty impacts on all firms equally and, thus, rather influences the choice between different markets. Second, firm-specific uncertainty captures the idea that firms are heterogeneous ex-post. Therefore, our setting is consistent with the view that firms' responses to changes in economic conditions vary because,

[7] Greater uncertainty can also come about by a change in the probability distribution. Any variation in a simple probability distribution would require an adjustment in cost levels as well, in order to hold constant expected costs. Thus, we can look at cost changes directly, which are essential for our findings.

[8] In our setting, it suffices if each firm learns about its own costs. Since the ex-post distribution of costs is known before entry, firms no longer face uncertainty once they have learned whether $c = c_l$ or $c = c_h$ holds. Any expenditure for establishing own costs could be added to (or incorporated into) entry costs, k, without qualitatively affecting the subsequent analysis.

for example, their workforces have differential abilities. Third, entry in models without uncertainty has been investigated using the sum of profits and consumer surplus as the welfare measure. In the case of uncertainty, expected magnitudes can be employed. As is well known, expected consumer surplus represents an adequate welfare indicator under restrictive assumptions only. We avoid the ensuing problem of how to interpret variations in expected consumer surplus by assuming that the ex-ante distribution of production costs is also realized ex-post, which is feasible for the case of firm-specific uncertainty. This simplifying assumption implies that marginal cost realizations are correlated across firms ex-post and represents a convenient modeling device.

Production costs of firm j, $j = 1, ..., n$, are given by $c_i q_{ji}$, $i = h, l$, with output being denoted by q_{ji}. The inverse demand function is linear and reads $p(Q) = 1 - Q$, where $Q = q_{ji} + Q_{-j} = 0.5\sum_{j=1}^{n}(q_{jl} + q_{jh})$ is aggregate output, and Q_{-j} is the total output manufactured by the competitors of firm j.

Profits of firm j in state i can be expressed as

$$\pi_{ji} = (1 - Q)q_{ji} - c_i q_{ji} - k. \tag{1}$$

As operating profits of high-cost firms are positive (see below), expected profits read

$$\pi_j^e = 0.5(\pi_{jl} + \pi_{jh}) = 0.5((1 - Q)(q_{jl} + q_{jh}) - c_l q_{jl} - c_h q_{jh}) - k. \tag{2}$$

Welfare W is defined as the sum of aggregate expected profits and consumer surplus

$$W = \sum_{j=1}^{n} \pi_j^e + 0.5Q^2. \tag{3}$$

We treat n as a continuous variable (see, inter alia, Delipalla and Keen, 1992, Seade, 1980) and distinguish two scenarios. In the market equilibrium, expected profits determine n, while in the second-best optimum, a social planner selects the number of firms, but not their type, to maximize W.[9] The sequence of decisions is as follows:

[9]See Mankiw and Whinston (1986), Perry (1984), Varian (1995), and Amir et al. (2014) who pursued this approach. von Weizsäcker (1980) and Suzumura and Kiyono (1987) derive the first-best outcome.

1a) Firms decide about market entry, i.e., whether to invest k or abstain from doing so.

or

1b) the social planner selects the number of firms which enter the market and invest k.

2) All firms learn about their own and the competitors' marginal production costs.

3) Firms simultaneously choose output.

We solve the aforementioned game by backward induction and focus on pure strategy equilibria. Because a firm's behavior is qualitatively the same, irrespective of how the number of competitors is determined, the analysis in 3) applies to the market equilibrium and the social planner's considerations.

In our setting with linear demand, profits, aggregate output and welfare can be formulated as functions of the number of firms, n, and the maximum surplus. Because uncertainty affects both n and the surplus, we subsequently state the main outcomes as functions of the number of firms, n, and the uncertainty parameter, ϵ, as well as of ϵ only.

4 Solution

4.1 Output Choices

In stage 3, firms maximize profits with respect to output, taking as given the number of firms, n, and marginal production costs, c_i. The first-order condition for firm j reads

$$\frac{d\pi_{ji}}{dq_{ji}} = -q_{ji} + 1 - Q - c_i = 0. \tag{4}$$

The second-order condition is fulfilled. Solving (4) for the best-response function of each type of firm, taking into account that 50% of all firms face high and the other 50% feature low production costs, and that firms with the same costs behave identically, we obtain $q_l = 1 - 0.5n(q_l + q_h) - c_l$ for a low-cost firm and an equivalent expression for a high-cost firm. Simplification yields $q_l = (1 - 0.5nq_h - c_l)/(1 + 0.5n)$ and $q_h = (1 - 0.5nq_l - c_h)/(1 + 0.5n)$: Combining these equations, we can express the output of each firm type for any number of firms n as

$$q_l(\epsilon, n) = \frac{1 - c + (1+n)\epsilon}{1+n}, \tag{5}$$
$$q_h(\epsilon, n) = \frac{1 - c - (1+n)\epsilon}{1+n}. \tag{6}$$

Using (5) and (6), we can calculate expected output per firm as

$$\tilde{q}(n) = 0.5(q_l + q_h) = \frac{1-c}{1+n}. \tag{7}$$

Since expected output per firm decreases in the number of competitors, n, there is business stealing. Moreover, uncertainty affects output only via the number of firms. Because all firms of the same type choose the same output, aggregate output is a function of n (from (5) and (6)), which, therefore, can be computed on the basis of profit-maximising choices, once the entry decisions have been observed:

$$Q(n) = \frac{n(1-c)}{1+n}. \tag{8}$$

Combining (4) and (8) clarifies that operating profits of a high-cost firm will be positive if the extent of uncertainty does not exceed a critical level implicitly defined by

$$1 - Q(n) - c - \epsilon = \frac{1 - c - \epsilon(1+n)}{1+n} > 0. \tag{9}$$

4.2 Market Equilibrium

To determine the market outcome, we first compute the equilibrium number of firms n^*. Rearranging (2) and utilizing the feature that all firms are ex-ante identical yields

$$\begin{aligned} \pi^e(\epsilon, n) &= 0.5\left((1 - Q(n))(q_l^*(\epsilon, n) + q_h^*(\epsilon, n)) - c_l q_l^*(\epsilon, n) - c_h q_h^*(\epsilon, n)\right) - k \\ &= \frac{(1-c)^2}{(1+n)^2} + \epsilon^2 - k. \end{aligned} \tag{10}$$

From $\pi^e(\epsilon, n) = 0$ we obtain (see, f.e., Etro, 2014 and Mas-Colell et al., 1995 for $\epsilon = 0$)

$$n^*(\epsilon) = \frac{1-c}{(k-\epsilon^2)^{0.5}} - 1. \tag{11}$$

We calculate equilibrium output per firm, $q_l^*(\epsilon)$ and $q_h^*(\epsilon)$, expected output, $\tilde{q}^*(\epsilon)$, and aggregate output, $Q^*(\epsilon)$, by inserting (11) into (5), (6), (7), and, (8), respectively:[10]

$$q_l^*(\epsilon) = (k-\epsilon^2)^{0.5} + \epsilon, \tag{12}$$

$$q_h^*(\epsilon) = (k-\epsilon^2)^{0.5} - \epsilon, \tag{13}$$

$$\tilde{q}^*(\epsilon) = (k-\epsilon^2)^{0.5}, \tag{14}$$

$$Q^*(\epsilon) = 1 - c - (k-\epsilon^2)^{0.5}. \tag{15}$$

Accordingly, welfare is given by

$$\begin{aligned} W(n^*(\epsilon)) = W^*(\epsilon) &= 0.5[Q^*(\epsilon)]^2 = 0.5(1-c-(k-\epsilon^2)^{0.5})^2 \\ &= 0.5\left[\frac{n^*(\epsilon)(1-c)}{1+n^*(\epsilon)}\right]^2 = 0.5(k-\epsilon^2)[n^*(\epsilon)]^2. \end{aligned} \tag{16}$$

4.3 Social Optimum

When selecting the (second-best) welfare-maximizing number of firms, the social planner takes into account output decisions of ex-ante identical firms and maximizes

$$W^e(n) = n\pi^e(\epsilon, n) + 0.5[Q(n)]^2. \tag{17}$$

[10] From (9) and (1), we observe that operating profits are positive if

$$1 - c - \epsilon(1 + n^*(\epsilon)) = (1-c)\left[1 - \frac{\epsilon}{(k-\epsilon^2)^{0.5}}\right] > 0.$$

This is tantamount to $k > 2\epsilon^2$. If the number of firms in a socially optimal situation is lower than in market equilibrium, any value of $n^*(\epsilon)$ which satisfies the constraint will also guarantee that the restriction holds if the social planner determines the number of firms.

The first-order condition reads

$$\frac{dW^e}{dn} = \pi^e(n,\epsilon) + \underbrace{n \underbrace{\frac{d\pi^e}{dn}}_{<0} + Q(n) \underbrace{\frac{dQ}{dn}}_{>0}}_{<0} = 0. \tag{18}$$

Using $d\pi^e/dn = -2(1-c)^2/(1+n)^3$, as well as $dQ/dn = (1-c)/(1+n)^2$, it is straightforward to establish $nd\pi^e/dn + Q(n)dQ/dn < 0$. Hence, the welfare-maximum is characterized by profitable firms. Inserting (8) and (10) into (18) shows that W^e is strictly concave in n. Rearranging the resulting expression and also using (11), we can calculate the socially optimal number of firms, $n^{opt}(\epsilon)$, as (see Etro, 2014 and Mas-Colell et al., 1995 for $\epsilon = 0$)

$$n^{opt}(\epsilon) = \left(\frac{1-c}{(k-\epsilon^2)^{0.5}}\right)^{2/3} - 1 = (1+n^*(\epsilon))^{2/3} - 1. \tag{19}$$

Inserting (19) into (7), (8) and (10) leads to expected output per firm, $\tilde{q}^{opt}(\epsilon)$, aggregate output, $Q^{opt}(\epsilon)$, and expected profits, $\pi^{eopt}(\epsilon)$, in the social optimum

$$\tilde{q}^{opt}(\epsilon) = (1-c)^{1/3}(k-\epsilon^2)^{1/3}, \tag{20}$$

$$Q^{opt}(\epsilon) = 1 - c - (1-c)^{1/3}(k-\epsilon^2)^{1/3}, \tag{21}$$

$$\pi^{eopt}(\epsilon) = (1-c)^{2/3}(k-\epsilon^2)^{2/3} + \epsilon^2 - k = (k-\epsilon^2)n^{opt}(\epsilon). \tag{22}$$

Finally, expected socially optimal welfare $W^{eopt}(\epsilon)$ can be calculated by substituting (19), (22) and (21) into the definition of welfare.

$$\begin{aligned} W^{eopt}(\epsilon) &= n^{opt}(\epsilon)\left[\left(\frac{1-c}{1+n^{opt}(\epsilon)}\right)^2 + \epsilon^2 - k\right] + 0.5\left(\frac{n^{opt}(\epsilon)(1-c)}{1+n^{opt}(\epsilon)}\right)^2 \\ &= 0.5(1-c)^2 + k - \epsilon^2 - 1.5(1-c)^{2/3}(k-\epsilon^2)^{2/3}. \end{aligned} \tag{23}$$

5 The Effects of Cost Uncertainty

5.1 Analytical Results

If there is business stealing, entry is excessive and output per firm is inefficiently low (see, e.g., Mankiw and Whinston, 1986). This excessive entry prediction assumes certain payoffs. In a first step, we will show that cost uncertainty does not invalidate the prediction. Second, we analyze how the endogenous variables change with cost uncertainty in market equilibrium and in social optimum and, more importantly, consider their relative variation. We will demonstrate that cost uncertainty exacerbates the excessive entry result, but can mitigate the adverse welfare consequences due to market power.

Comparing the number of firms (cf. (11) and (19)), expected output ((14) and (20)), and aggregate output in the equilibrium and social optimum (i.e., (15) and (21)), we find:

Proposition 1.

The excessive entry prediction also results in the presence of cost uncertainty, i.e. for $\epsilon > 0$. Specifically, entry is excessive, $n^*(\epsilon) > n^{opt}(\epsilon)\ \forall\epsilon$, expected output per firm is too low, $\tilde{q}^*(\epsilon) < \tilde{q}^{opt}(\epsilon)\ \forall\epsilon$, and aggregate output is too high, $Q^*(\epsilon) > Q^{opt}(\epsilon)\ \forall\epsilon$.

Proof 1.

See text. □

Intuitively, a firm entering the market does not internalize the business-stealing externality. Moreover, expected output per firm is unaffected by the presence of cost uncertainty, for a given number of firms, and, thus, declines with the number of competitors. Therefore, the output externality continues to exist. Effectively, cost uncertainty does not constitute another externality which could reverse the business-stealing effect.[11]

Proposition 1 holds, irrespective of the level of fixed costs, k. This is in contrast to settings with vertical relationships in which fixed costs can affect the possibility of insufficient entry (see Mukherjee, 2009). The difference arises because fixed costs influence the extent to which the business-creation impact (Ghosh and Morita, 2007a), which occurs

[11]Relative to a first-best setting in which the social planner additionally determines output per firm, a further source of welfare loss arises due to uncertainty. For a given level of aggregate output, shifting production from high-cost to low-cost firms, ceteris paribus, raises welfare. The extreme case of such shifting is the closure of high-cost firms, which we consider in Section 6.3.

if entry by downstream firms enhances payoffs of upstream providers of inputs, counteracts the business-stealing externality. There is no such mechanism present in our setting.

The next proposition states how uncertainty alters the market equilibrium:

Proposition 2.

In market equilibrium, an increase in uncertainty, i.e. a rise in ϵ,

(i) raises the number of firms $n^*(\epsilon)$,

(ii) reduces expected output per firm $\tilde{q}^*(\epsilon)$,

(iii) raises total output $Q^*(\epsilon)$, and

(iv) raises welfare $W^*(\epsilon)$.

Proof 2. See Appendix A.1. □

An increase in ϵ raises the spread between low and high marginal costs while leaving its mean unchanged. Expected total production costs $0.5(c_h q_h + c_l q_l)$ decline since the cost reduction of low-cost firms dominates the increase of high-cost firms. In addition, a change in ϵ has no (direct) effect on expected production. Consequently, expected profits unambiguously increase, which gives more firms an incentive to enter the market, i.e. n^* increases.[12] The new entrants steal business from incumbents, such that expected output per firm declines. Aggregate output increases, showing that the rise in n^* dominates the fall of $\tilde{q}^*$.[13] Given the rise of Q^*, welfare increases as well.

We can compare Propositions 1 and 2 with the findings by Deo and Corbett (2009), who analyze yield uncertainty, i.e., a situation in which a given level of inputs leads to an ex-ante unknown quantitative yield and, thus, sales. Output cannot be adjusted ex-post, after information about the saleable quantity has become public information. In addition, marginal costs are subject to uncertainty. These features of the model by Deo and Corbett

[12] Elberfeld and Nti (2004) allow for costly investments which may reduce marginal costs and analyze the incentives of an exogenously given number of Cournot oligopolists to undertake this kind of investment. They show that such cost uncertainty raises expected profits since firms can respond to the realization of costs. This finding is consistent with our analysis with an endogenously determined number of competitors. Haruna (1992) also derives a positive effect of a mean-preserving rise in factor-price uncertainty on the number of firms in a competitive environment in which entrants have no market power.

[13] Comparing these findings with those by Corchón and Fradera (2002) indicates that the consequences of cost uncertainty are qualitatively the same as those resulting from a reduction in marginal costs.

(2009) imply that the production level, which maximizes profits ex-post, is only chosen correctly ex-ante by chance and that expected output per firm falls with uncertainty, for a given number of competitors. Furthermore, uncertainty makes entry less attractive. Therefore, aggregate output and consumer surplus in market equilibrium decline with uncertainty, in contrast to the findings summarized in Proposition 2. Finally, Deo and Corbett (2009) show that yield uncertainty mitigates the business-stealing externality and can give rise to insufficient entry if the uncertainty effect is strong enough. Accordingly, yield uncertainty to which firms cannot respond after it has been resolved may have fundamentally different effects than the ex-ante cost uncertainty that we consider.

We next turn to the consequences of uncertainty for the socially optimal outcome. They are summarized in

Proposition 3.

In the (second-best) social optimum, an increase in uncertainty, i.e. a rise in ϵ,

(i) raises the number of firms $n^{opt}(\epsilon)$,

(ii) reduces expected output per firm $\tilde{q}^{opt}(\epsilon)$,

(iii) raises total output $Q^{opt}(\epsilon)$,

(iv) reduces expected profits $\pi^{eopt}(\epsilon)$, and

(v) raises welfare $W^{opt}(\epsilon)$.

Proof 3. See Appendix A.1. □

An increase in ϵ raises expected profits, for a given number of firms, while aggregate output varies with uncertainty only via the number of firms. Moreover, the derivatives $d\pi^e/dn$ and dQ/dn in (18) do not directly depend on ϵ. Thus, the welfare-maximizing number of firms increases with cost uncertainty. Accordingly, expected output per firm declines while aggregate output increases. Regarding π^{eopt}, two effects work in opposite directions. On the one hand, the decline in expected production costs increases expected profits. On the other hand, expected profits fall due to the higher number of competitors. Because the latter effect exceeds the former π^{eopt} shrinks. In sum, welfare increases since the impact on aggregate output dominates the decline in expected profits.

Comparing Propositions 2 and 3, we can note that all variables change in the same direction in market equilibrium and in a socially optimal situation. Because the social planner internalizes the business-stealing externality while firms do not, we, however, can expect the quantitative impact of these changes to differ. In how far this expectation is justified is stated in our final and central

Proposition 4.

An increase in uncertainty, i.e. a rise in ϵ,

(i) raises the difference between $n^*(\epsilon)$ and $n^{opt}(\epsilon)$, making the excessive entry inefficiency more pronounced,

(ii) raises the difference between $\tilde{q}^{opt}(\epsilon)$ and $\tilde{q}^*(\epsilon)$ if $n^* < 2.375$ and lowers this difference, making the insufficient output per firm inefficiency less pronounced, if $n^* > 2.375$,

(iii) raises the difference between $Q^*(\epsilon)$ and $Q^{opt}(\epsilon)$ if $n^* < 2.375$, and reduces this difference, that is, mitigates the excessive aggregate output result, if $n^* > 2.375$ and

(iv) raises the difference between $W^{opt}(\epsilon)$ and $W^*(\epsilon)$, if $n^* < 1 + \sqrt{5} \approx 3.24$ and reduces the welfare loss due to oligopoly if $n^* > 1 + \sqrt{5}$.

Proof 4. See Appendix A.1. □

To provide intuition, note that firms and the social planner react differently to the reduction of expected production costs (caused by higher uncertainty). While the planner is aware of the resulting consequences for aggregate supply, firms ignore the implications for their competitors. From (10) we see that the marginal profit effect of uncertainty is independent of the number of firms, n. Moreover, profits decline in n at a decreasing rate. Since there is excessive entry, a given reduction in expected costs, due to more pronounced uncertainty, results in a greater increase in the number of firms in market equilibrium than in the second-best socially optimal set-up.

From equation (7), we can further observe that the effect of uncertainty on expected output per firm arises because the number of firms changes. The strength of this impact depends, first, on how many additional firms enter and, second, on the stock of firms. Because the number of firms rises more strongly in market equilibrium, the first effect implies that the insufficient output outcome becomes more pronounced. Second, a given increase in the number of firms reduces output per firm by a smaller amount the more firms there are. Since the number of firms is higher in market equilibrium, this second effect, ceteris paribus, implies that the inefficiency with respect to output per firm becomes less pronounced. The more firms compete, the stronger this second impact becomes, relative to the first one, because the absolute difference in the number of firms is larger. Therefore, the greater the number of firms is, the more likely it is that the insufficiency of output per firm declines with cost uncertainty. Given linear demand and cost functions, the threshold, at which the second effect dominates, can be computed explicitly as $n^* = 2.375$ ($n^{opt} = (1 + n^*)^{2/3} - 1 = 1.25$). Using (11) and (19), this number can alternatively be expressed in terms of cost parameters, that is, $(1 - c)/((k - \epsilon^2)^{0.5}) = 3.375$.

Aggregate output varies with uncertainty only via expected output per firm (cf. (15) and (21)). In consequence, the same restriction on their number also affects the assessment of the variations in aggregate output. Hence, if the difference between optimal expected output per firm and the level chosen in market equilibrium falls, that is, if the number of firms in equilibrium is sufficiently high, the positive difference between aggregate output in market equilibrium and the optimal amount will also shrink with uncertainty.

Proposition 4, finally, asserts that the welfare loss due to excessive entry will decline with uncertainty if the number of firms exceeds a critical level, $n^* = 1 + \sqrt{5}$. This number is higher than the one, which is required for the aggregate output inefficiency to decline, $n^* = 2.375$. As stated above, expected profits per firm fall with greater uncertainty in a socially optimal situation. Moreover, the increase in the number of firms raises aggregate profits. In a socially optimal situation, the net profit impact on welfare is positive. Since profits are zero in market equilibrium, the profit effect of greater uncertainty raises the welfare loss due to oligopoly. At the same time, aggregate output rises and increases welfare via the growth in consumer surplus in equilibrium and the social optimum. The resulting change in output due to greater uncertainty is the same if $n^* = 2.375$. Because output is higher in market equilibrium and is concave in the number of firms, n, a given rise in aggregate output has a smaller positive welfare impact in market equilibrium than in a socially optimal situation. Hence, at $n^* = 2.375$ greater uncertainty raises the welfare loss, taking the profit and consumer surplus impact into account. If the number of firms is higher, aggregate output in market equilibrium increases by less than in a socially optimal situation (cf. Proposition 4, part (*iii*)), and welfare in market equilibrium rises more strongly than if the social planner determined entry. If, therefore, the number of firms in equilibrium exceeds $n^* = 1 + \sqrt{5} = 3.24$ ($n^{opt} = 1.62$), greater cost uncertainty reduces the welfare loss due to oligopoly.

Since the number of firms in market equilibrium declines in fixed enty costs, k, (cf. (11)), greater uncertainty is less likely to reduce the welfare loss from excessive entry, the higher k is. This is because the excess number of firms rises with entry costs. Hence, the profit (output) effect tends to have a greater (lower) importance.

5.2 Numerical Example

To get an impression of the quantitative importance of our results, we subsequently solve the model numerically. We set $k = 0.06$ and $c = 0.2$, which implies that the relation between a firm's revenues and market entry costs is about 1.6. Table 1 illustrates outcomes in equilibrium and in social optimum. In both cases, we calculate the percentage change of the respective variable if ϵ increases from 0 (no uncertainty) to the value 0.17, which implies relatively high uncertainty, without violating the constraints stated in Section 3. For instance, the (percentage) effect on the number of firms, Δn, is computed as

$$\Delta n = \frac{n(\epsilon = 0.17) - n(\epsilon = 0)}{n(\epsilon = 0)} \times 100 \qquad \text{for } n = n^* \text{ and } n = n^{opt}.$$

The first row in Table 1 reflects Proposition 2, while the second row illustrates the findings stated in Proposition 3. The values indicate the substantial quantitative effects of uncertainty on the number of firms, aggregate output and welfare. Since welfare is higher in a socially optimal situation than in market equilibrium by definition, while uncertainty raises welfare by more in the latter than in the former (cf. the last column in Table 1), the welfare loss due to oligopoly may change in either direction.

In Table 2, we, therefore, compare the differential effects on the equilibrium and socially optimal outcomes. We depict how the difference, e.g., in the number of firms,

Table 1: The Effects of Uncertainty I

	Δn	$\Delta\tilde{q}$	ΔQ	$\Delta\pi$	ΔW
Equilibrium	56	-28	12	0	26
Social Optimum	45	-20	16	-25	23

Note: Percentage changes rounded to full numbers

changes when moving from a situation with known costs ($\epsilon = 0$) to uncertain settings ($\epsilon > 0$).[14] From (11) and (19), we know that the number of firms is uniquely associated with the degree of uncertainty and rises with ϵ. Thus, the critical values relating to the number of firms stated in Proposition 4 can also be expressed in terms of ϵ. $n^* = 2.375$ holds if $\epsilon = \epsilon_1^{crit} \approx 0.062$ and $n^* = 1 + \sqrt{5}$ holds if $\epsilon = \epsilon_2^{crit} \approx 0.16$. Table 2 mimics our findings in Proposition 4. In particular, uncertainty increases excessive entry.[15]

Table 2: The Effects of Uncertainty II

	$\epsilon = 0$	$\epsilon_1^{crit} \approx 0.062$	$\epsilon_2^{crit} \approx 0.16$	$\epsilon = 0.17$
$n^*(\epsilon) - n^{opt}(\epsilon)$	1.0647	1.125	1.61803	1.79603
$\tilde{q}^{opt}(\epsilon) - \tilde{q}^*(\epsilon)$	0.118475	0.118519	0.116718	0.115581
$Q^*(\epsilon) - Q^{opt}(\epsilon)$	0.118475	0.118519	0.116718	0.115581
$W^{opt}(\epsilon) - W^*(\epsilon)$	0.0278435	0.0280933	0.0288544	0.0287941

Notes: At $\epsilon = \epsilon_1^{crit} = 0.062$, $n^* = 2.375$, while $\epsilon = \epsilon_2^{crit} = 0.16$ implies that $n^* = 1 + \sqrt{5}$.

While the rise in the number of firms suggests a massive intensification of competition in market equilibrium, a look, for example, at the Lerner-index, $L = (p(Q) - c)/p(Q) = 1 - c/(1-Q)$ provides a more nuanced picture.[16] In our setting, the Lerner-index, assuming a monopoly ($n = 1$), is $L = 0.67$ and the resulting price-cost mark-up, $\mu = 1/(1 - L)$, equals 3. In a free-entry oligopoly with certainty ($\epsilon = 0$), the Lerner-index takes a value of $L = 0.55$ ($\mu = 2.2$) in market equilibrium, declining to $L = 0.47$ ($\mu = 1.89$) if uncertainty is high ($\epsilon = 0.17$). These numbers can be compared to estimates of mark-ups for the United States, which range between 1.1 and 1.7 (De Loecker et al., 2020, Hall, 2018). As such, the degree of competition in our model may be considered as relatively moderate,

[14] Our results are robust with respect to variations in entry costs k.

[15] The difference between expected output per firm, $\tilde{q}^{opt}(\epsilon) - \tilde{q}^*(\epsilon)$, and aggregate output, $Q^*(\epsilon) - Q^{opt}(\epsilon)$, is the same for a given level of ϵ. This is due to the assumption of linear costs, as the analysis of quadratic costs in Appendix A.2.2 clarifies.

[16] Other concentration measures, such as the 4-firm concentration ratio or the Herfindahl-Hirschman-index, are not informative in our setting with only two types of firms. This is one reason why the empirical evidence on uncertainty and output market concentration (see, for example, Ghosal, 1995, 2010) cannot help to evaluate the theoretical findings. Moreover, these and other contributions, such as by Ghosal and Loungani (1996), which look at the number of firms, employ measures of uncertainty which are difficult to relate to ϵ.

even in case of high uncertainty.

6 Extensions

Our main result has been derived under a set of simplifying assumptions. In order to evaluate the robustness of our findings, we subsequently analyze four modifications.

6.1 Comprehensive Uncertainty

As a first extension, we assume that all firms face the same cost realization ex-post. In consequence, aggregate output is uncertain ex-ante and can ex-post be either high or low. It could be surmised that comprehensive uncertainty affects oligopoly outcomes differently than firm-specific uncertainty because the resulting production level will be different. Moreover, empirical evidence on the impact of uncertainty on investment indicates such differences (Baum et al., 2008, Henley et al., 2003).

We can show that the market equilibrium is characterized by excessive entry, insufficient expected output per firm, and excessive aggregate output (see Appendix A.2.1). Hence, Proposition 1 also holds in the case of comprehensive marginal cost uncertainty. In addition, the market equilibrium changes in the same way with greater uncertainty as described in Proposition 2. If the social planner determines entry, we obtain the same predictions as summarized in Proposition 3, with the exception of the change in profits. Comparing the market equilibrium and the socially optimal outcome, greater uncertainty unambiguously aggravates excessive entry, but mitigates the insufficient output per firm and excessive aggregate output externalities if $n^* > 2.375$ holds. These are the same findings as stated in Proposition 4.

As indicated in the introduction, expected consumer surplus is a problematic measure of consumer welfare (see, inter alia, Creane, 2007, Deo and Corbett, 2009, Schlee, 2008, Silvers, 2018). Nonetheless, it is reassuring that the predictions relating to firm-specific uncertainty carry over to comprehensive cost uncertainty. The only decisive difference is that expected profits rise with comprehensive cost uncertainty, given the socially optimal number of firms, whereas they decline with firm-specific uncertainty. The difference arises because the profit-reducing effect of more competitors is less pronounced in case of comprehensive uncertainty. If profits rise, welfare in the social optimum will increase with greater comprehensive uncertainty more strongly than in market equilibrium.

6.2 Quadratic Cost Function

One might further question whether our results for firm-specific uncertainty rely on the linear cost function. On the one hand, there also is excessive entry if costs are quadratic (see, f.e., von Weizsäcker, 1980). On the other hand, the impact of cost uncertainty on firm choices becomes more pronounced if the cost function is strictly convex because the gains from output adjustments rise. While these arguments suggest that the effect of uncertainty on the excessive entry prediction will not be altered qualitatively, it is less obvious whether other outcomes change more strongly in market equilibrium or the socially optimal outcome.

In order to answer the above (implicit) question, we presume that variable costs are given by $0.5c_iq_i^2$, $i = h, l$. Such modification substantially increases the complexity of the model. Nonetheless, we can establish a number of results analytically (see Appendix A.2.2).[17] First, the excessive entry prediction also holds (cf. Proposition 1). Second, uncertainty increases the number of firms in market equilibrium if entry costs exceed a critical value. While expected output per firm declines, all other results summarized in Proposition 2 continue to hold. Similarly, if entry costs are not too low, greater uncertainty raises the socially optimal number of firms, total output and welfare, as predicted in Proposition 3. In order to determine the effects on the differential between the market equilibrium and the social optimum, we rely on a numerical solution. Using the same parameter values as in Section 5.2, we observe that uncertainty aggravates excessive entry, the excessive aggregate output result, and the welfare loss due to oligopoly. These are the same findings as summarized in Table 2. However, we obtain no numerical evidence for a non-linearity of the welfare difference, as in the case of a linear cost function.

[17] de Pinto and Goerke (2018) provide a more detailed derivation of the setting with quadratic costs.

6.3 The Role of Firm Closures

Our analysis so far is based on the assumption that all firms which enter the market produce a positive quantity. However, the variance of marginal costs may be so pronounced that high-cost firms prefer to close down after production costs have become known and before output decisions are made.[18] To analyze this possibility, we assume that a firm can enter the market at costs k, where $k < \epsilon^2$ holds. A firm, which faces high marginal costs, $c_h = c + \epsilon$, would therefore not only realize negative profits, but also raise the loss by producing a positive amount. If exit, that is, the production of zero output, is feasible at no additional costs, all high-cost firms refrain from taking up production. All low-cost firms remain in the market and adjust their output levels to the exit decisions.

In such a setting, greater uncertainty, that is a further increase in ϵ, raises profits of low-cost firms, while high-costs firms continue to have operating profits of zero. Thus, uncertainty enhances profits in market equilibrium (for a given number of firms; see Février and Linnemer, 2004, Corollary 3.2) and makes entry more attractive.

To preserve comparability between the market equilibrium and the socially optimal outcome, we assume that half of the entrants exit if the social planner determines entry. Accordingly, all $0.5n$ firms producing a positive quantity face low marginal costs, $c_l = c-\epsilon$. Given this modification, we show in Appendix A.2.3 that uncertainty raises the optimal number of firms. Moreover, there is excessive entry, and an increase in uncertainty makes the excessive entry inefficiency more pronounced.

This result contrasts with the finding by Creane (2007) who demonstrates that there may be insufficient entry if paying entry costs does not guarantee a positive output level. The differences arises because output per firm is fixed in the set-up by Creane (2007) and the number of firms cannot adjust continuously. Importantly, if output per firm is constant, the business-stealing externality is absent at the margin and the main mechanism inducing excessive entry in our modeling set-up does not apply.

We, furthermore, show (in Appendix A.2.3) that Propositions 2 and 3 hold true. There are two exceptions: As there are only low-cost firms, output per firm rises with uncertainty because an increase in ϵ is equivalent to lower marginal costs for firms that do not exit.

[18]We are extremely grateful to an anonymous referee for suggesting this extension.

Moreover, this implies that expected profits per firm rise in the social optimum.

Observe, finally, that the above line of argument is based on the assumption that high-cost firms exit the market, irrespective of the extent of uncertainty (see Appendix A.2.3). Alternatively, we could compare a situation in which uncertainty is sufficiently low, such that high-cost firms still produce a positive quantity, with a situation with greater uncertainty in which these firms exit the market. Because greater marginal cost uncertainty aggravates excessive entry, irrespective of whether exit occurs or not, our main result would also apply if the increase in uncertainty induces firms to exit the market.

6.4 Integer Constraint and Uncorrelated Probabilities

In our forth extension, we depart from the assumption that the number of firms is a continuous variable and impose an integer constraint. To analyze whether greater uncertainty aggravates the excessive entry problem (cf. Proposition 4), we utilize the numerical example of Section 5.2. Moreover, when imposing the integer constraint, we can straightforwardly investigate a situation in which cost realizations are random and uncorrelated across firms ex-post.[19] Therefore, the entire binomial distribution is relevant for entry decisions. This extension allows us to evaluate whether a combination of firm-specific and comprehensive uncertainty (see Proposition 4 and Section 6.1) gives rise to the same effects concerning excessive entry as each type of uncertainty on its own.

6.4.1 Integer Constraint

The number of firms in market equilibrium, $n^{*,int}(\epsilon)$, is determined by a situation in which expected profits, as given by (10), are still positive for $n^{*,int}(\epsilon)$ but become negative for $n^{*,int}(\epsilon)+1$. The socially optimal number of firms, $n^{opt,int}(\epsilon)$, is defined by $W^e(\epsilon, n^{opt,int}(\epsilon)-1) < W^e(\epsilon, n^{opt,int}(\epsilon)) > W^e(\epsilon, n^{opt,int}(\epsilon)+1)$, where expected welfare is implicitly defined in the first line of (23) and strictly concave in n.

Table 3 shows that uncertainty increases entry, irrespective of who decides about it. The relatively stronger increase in market equilibrium, as established for a setting without

[19]This alternative modeling approach was suggested by an anonymous referee who also provided detailed calculations, which we partly replicate in Appendix A.2.4. We are deeply indebted to the referee for this constructive suggestion and the generosity.

Table 3: Number of Firms and Welfare Loss in Base Model with Integer Constraint ($c = 0.2$, $k = 0.06$)

	$\epsilon = 0$	$\epsilon = 0.05$	$\epsilon = 0.1$	$\epsilon = 0.15$	$\epsilon = 0.17$
$n^{*,int}(\epsilon)$	2	2	2	3	3
$n^{opt,int}(\epsilon)$	1	1	1	2	2
$n^{*,int}(\epsilon) - n^{opt,int}(\epsilon)$	1	1	1	1	1
$W^e(\epsilon, n^{opt,int}(\epsilon)) - W^e(\epsilon, n^{*,int}(\epsilon))$	0.016	0.016	0.006	0.021	0.015

integer constraint, is not reflected in Table 3. This is the case because the number of firms is small. If fixed costs are lower ($k = 0.03$; not depicted in Table 3), the increase in the number of firms and excessive entry become more pronounced. Hence, we obtain no evidence that the main ingredient of Proposition 4 is due to the simplification that the number of firms can vary continuously. Finally, in Table 3, the welfare loss due to free entry, $W^e(\epsilon, n^{opt,int}(\epsilon)) - W^e(\epsilon, n^{*,int}(\epsilon))$, does not show the inverted u-shaped relationship with uncertainty arising in the base model (see Proposition 4). This confirms the impression from Table 2 that the magnitude of the welfare loss due to oligopoly varies with uncertainty, but may be moderate.

6.4.2 Marginal Costs as Random Variable

Suppose next that the probability of having high marginal costs represents a binomial random variable, such that the aggregate quantity cannot be anticipated ex-ante, that is, when deciding about entry. The main difference to Section 3 is that the ex-post distribution of marginal costs may deviate substantially from the ex-ante distribution. Since cost realizations are public knowledge ex-post, firms can utilize this feature to adjust output. Because this is true for the market equilibrium and the socially optimal setting, a priori it is not obvious how comprehensive uncertainty, in addition to firm-specific uncertainty considered in the base model, affects the excessive entry prediction.

While expected profits and welfare can be computed explicitly (see Appendix A.2.4), we use the numerical example to investigate the effects of greater uncertainty. Table 4 depicts the (integer) number of firms in market equilibrium, the corresponding second-best optimal outcome, the number of excess entrants, and the welfare loss due to oligopoly for two values of fixed costs.

Table 4: Number of Firms and Welfare Loss in a Model with Integer Constraint and Uncorrelated Costs ($c = 0.2$)

	$\epsilon = 0$	$\epsilon = 0.05$	$\epsilon = 0.1$	$\epsilon = 0.15$	$\epsilon = 0.17$
$k = 0.06$					
$n^{*,int}(\epsilon)$	2	2	2	2	2
$n^{opt,int}(\epsilon)$	1	1	1	2	2
$n^{*,int}(\epsilon) - n^{opt,int}(\epsilon)$	1	1	1	0	0
$W^e(\epsilon, n^{opt,int}(\epsilon)) - W^e(\epsilon, n^{*,int}(\epsilon))$	0.016	0.013	0.007	0	0
$k = 0.03$					
$n^{*,int}(\epsilon)$	3	3	4	6	12
$n^{opt,int}(\epsilon)$	2	2	2	3	6
$n^{*,int}(\epsilon) - n^{opt,int}(\epsilon)$	1	1	2	3	6
$W^e(\epsilon, n^{opt,int}(\epsilon)) - W^e(\epsilon, n^{*,int}(\epsilon))$	0.014	0.012	0.018	0.012	0.003

Table 4 demonstrates, first, that uncertainty hardly affects the number of firms if the fixed costs of entry, k, are relatively high, while the extent of excessive entry is constant or even diminishes. If the fixed costs of entry are lower the number of firms increases substantially with uncertainty, and the same is true concerning excessive entry. These differences are due to the fact that the number of firms in the absence of uncertainty is the lower, the higher fixed costs of entry are. Furthermore, we know from the proof of Proposition 4 (see (11), (A.1), and (A.8) in Appendix A.1) that a given increase in uncertainty has a smaller impact on excessive entry the higher the fixed costs are. Therefore, a given percentage increase in the number of competitors is less likely to cross the integer hurdle. Table 4 furthermore indicates that the welfare loss due to excessive entry may not be concave in uncertainty, as in the base model, if marginal costs are uncorrelated. However, the computations do not indicate whether this is due to the integer constraint (as Table 3 suggests) or the specification of uncertainty.

7 Conclusion

In this paper, we analyze the effects of ex-ante cost uncertainty on the welfare implications of oligopoly and on the excessive entry prediction. To that end, we consider a homogeneous Cournot oligopoly model with costly market entry. In our main specification, production costs are linear. Marginal costs can be either low or high and their ex-post distribution is known ex-ante, that is, prior to entry. Thus, cost realizations are correlated across firms. Upon entry, costs become public knowledge and firms decide about output. We distinguish two scenarios. In the market equilibrium, the number of firms is determined by a zero-profit condition in expected terms. In the second-best optimum, a social planner sets the number of firms to maximize expected welfare. As robustness checks, we consider comprehensive uncertainty (Rob I), i.e. ex-post all firms face the same cost realization, quadratic costs (Rob II), and allow for firm closures (Rob III). In addition, we analyze the implications of the integer constraint and the consequences of uncorrelated cost realizations within such set-up.

Table 5 summarizes our findings for the base model and those extensions in which the number of firms varies continuously. Uncertainty aggravates excessive entry, and the welfare gap widens in all robustness checks. The analysis of the integer constraint shows that it may mitigate the impact of uncertainty on excessive entry.

Table 5: Summary of our Results

	Main	Rob I	Rob II	Rob III
$n^*(\epsilon) - n^{opt}(\epsilon)$	increases	increases	increases	increases
$\tilde{q}^{opt}(\epsilon) - \tilde{q}^*(\epsilon)$	increases if $n^* < 2.375$ decreases if $n^* > 2.375$	increases if $n^* < 2.375$ decreases if $n^* > 2.375$	increases	increases
$W^{opt}(\epsilon) - W^*(\epsilon)$	increases if $n^* < 1 + \sqrt{5}$ decreases if $n^* > 1 + \sqrt{5}$	increases	increases	increases

Notes: The column "Main" refers to our main specification with firm-specific uncertainty and linear costs. The columns "Rob I", "Rob II" and "Rob III" indicate the cases of comprehensive uncertainty, quadratic costs and firm closures, respectively.

The investigation is of great relevance because oligopolies are pervasive in many sectors, and cost uncertainty has become more pronounced for a variety of reasons, such as globalization, political instability, the rise in the use of information and communication technologies, climate change, and, recently, global impediments to health. Our findings highlight that policies which aim to regulate oligopolies (or try to avoid their formation) may become more relevant from a welfare perspective, the less predictable market success is. Furthermore, prior to market interventions, policy makers have to carefully evaluate the market environment, in particular with respect to the level of uncertainty. This is the case since the optimal number of firms can vary substantially with cost uncertainty.

A Appendix

A.1 Proofs of Proposition 2, 3 and 4

To prove Proposition 2, we differentiate (11), (14), and (16)

$$\frac{dn^*}{d\epsilon} = \frac{(1-c)\epsilon}{(k-\epsilon^2)^{1.5}} > 0, \tag{A.1}$$

$$\frac{d\tilde{q}^*}{d\epsilon} = -\epsilon(k-\epsilon^2)^{-0.5} = -\frac{dQ^*}{d\epsilon} < 0, \tag{A.2}$$

$$\frac{dW^*}{d\epsilon} = \epsilon n^* > 0. \tag{A.3}$$

Differentiating (19), (20), (21), (22) and (23), proves Proposition 3.

$$\frac{dn^{opt}}{d\epsilon} = \frac{2}{3}(1+n^*)^{-1/3}\frac{dn^*}{d\epsilon} > 0 \tag{A.4}$$

$$\frac{d\tilde{q}^{opt}}{d\epsilon} = -\frac{2}{3}(1-c)^{1/3}\epsilon(k-\epsilon^2)^{-2/3} = -\frac{dQ^{opt}}{d\epsilon} < 0 \tag{A.5}$$

$$\frac{d\pi^{eopt}}{d\epsilon} = \frac{2}{3}\epsilon(1-2n^{opt}) < 0 \tag{A.6}$$

$$\frac{dW^{opt}}{d\epsilon} = 2\epsilon n^{opt} > 0 \tag{A.7}$$

Using (A.1) and (A.4), (A.2) and (A.5), we find

$$\frac{dn^*}{d\epsilon} - \frac{dn^{opt}}{d\epsilon} = \left(1 - \frac{2}{3}(1+n^*)^{-1/3}\right)\frac{dn^*}{d\epsilon} > 0, \tag{A.8}$$

$$\frac{d\tilde{q}^{opt}}{d\epsilon} - \frac{d\tilde{q}^*}{d\epsilon} = \frac{dQ^*}{d\epsilon} - \frac{dQ^{opt}}{d\epsilon} \quad = \epsilon(k-\epsilon^2)^{-0.5}\left(1 - \frac{2}{3}(1+n^*)^{1/3}\right), \tag{A.9}$$

which is negative if $n^* > 2.375$ ($n^{opt} > 1.25$). From (A.3) and (A.7), we obtain

$$\frac{dW^{opt}}{d\epsilon} - \frac{dW^*}{d\epsilon} = \epsilon\left(2(1+n^*)^{2/3} - 2 - n^*\right). \tag{A.10}$$

(A.10) is positive for $n^* = 1$, zero for $n^* = 1+\sqrt{5}$ and strictly concave in the number of firms. This proves Proposition 4.

A.2 Extensions

A.2.1 Comprehensive Uncertainty

Aggregate output, demand and profits per firm are given by $Q_i = nq_i$, $p(Q_i) = 1 - Q_i$ and $\pi_i = (1 - Q_i)q_i - c_iq_i - k$, respectively, with $i \in h, l$. Ex-ante, i.e. before costs are realized, expected output per firm, expected profits, and expected welfare read

$$q^e = 0.5(q_h + q_l), \tag{A.11}$$

$$\pi^e = 0.5(\pi_h + \pi_l), \tag{A.12}$$

$$W^e = 0.5(n\pi_h + 0.5Q_h^2 + n\pi_l + 0.5Q_l^2) = n\pi^e + 0.25(Q_h^2 + Q_l^2). \tag{A.13}$$

Market equilibrium is derived as described in the base model. Using the best-response function and the free-entry condition, we obtain

$$n^*(\epsilon) = \left(\frac{(1-c)^2 + \epsilon^2}{k}\right)^{0.5} - 1, \tag{A.14}$$

$$q^{e*}(\epsilon) = \frac{1-c}{1+n^*(\epsilon)}, \tag{A.15}$$

$$W^{e*}(\epsilon) = 0.5\left(\frac{n^*(\epsilon)}{1+n^*(\epsilon)}\right)^2((1-c)^2 + \epsilon^2) = 0.5[n^*(\epsilon)]^2k. \tag{A.16}$$

Differentiating these equations with respect to ϵ, we can show that $dn^*/d\epsilon > 0$. This implies that $dq^{e*}/d\epsilon < 0$, $dQ^{e*}/d\epsilon > 0$ and $dW^{e*}/d\epsilon > 0$.

The social planner maximizes $W^e(n) = n\pi^e(n, \epsilon) + 0.25([Q_h(n, \epsilon)]^2 + [Q_l(n, \epsilon)]^2)$ subject to

$$\pi^e(n, \epsilon) = \frac{(1-c)^2 + \epsilon^2}{(1+n)^2} - k, \tag{A.17}$$

$$[Q_h(n, \epsilon)]^2 + [Q_l(n, \epsilon)]^2 = 2\left(\frac{n}{1+n}\right)^2((1-c)^2 + \epsilon^2). \tag{A.18}$$

This results in

$$n^{opt}(\epsilon) = \left(\frac{(1-c)^2+\epsilon^2}{k}\right)^{1/3} - 1 = (1+n^*(\epsilon))^{2/3} - 1 < n^*(\epsilon), \tag{A.19}$$

$$q^{eopt}(\epsilon) = \frac{1-c}{1+n^{opt}(\epsilon)}, \tag{A.20}$$

$$\pi^{eopt}(\epsilon) = \frac{(1-c)^2+\epsilon^2}{(1+n^{opt}(\epsilon))^2} - k = kn^{opt}(\epsilon), \tag{A.21}$$

$$[Q_h(\epsilon)]^2 + [Q_l(\epsilon)]^2 = 2k[n^{opt}(\epsilon)]^2(1+n^{opt}(\epsilon)), \tag{A.22}$$

$$W^{eopt}(\epsilon) = n^{opt}(\epsilon)\pi^{eopt}(\epsilon) + 0.25([Q_h^{opt}(\epsilon)]^2 + [Q_l^{opt}(\epsilon)]^2). \tag{A.23}$$

Differentiating with respect to ϵ yields $dn^{opt}/d\epsilon > 0$, $dq^{eopt}/d\epsilon < 0$, $dQ^{eopt}/d\epsilon > 0$, $d\pi^{eopt}/d\epsilon > 0$ and $dW^{eopt}/d\epsilon > 0$.

Comparing market equilibrium with the social optimum, we find

$$\frac{dn^*}{d\epsilon} - \frac{dn^{opt}}{d\epsilon} > 0, \tag{A.24}$$

$$\frac{dq^{eopt}}{d\epsilon} - \frac{dq^{e*}}{d\epsilon} = \frac{dQ^{e*}}{d\epsilon} - \frac{dQ^{eopt}}{d\epsilon} = \left(1 - \frac{2}{3}(1+n^*)^{1/3}\right)\frac{1-c}{(1+n^*)^2}\frac{dn^*}{d\epsilon} < 0 \quad \text{if } n^* > 2.375, \tag{A.25}$$

$$\frac{dW^{eopt}}{d\epsilon} - \frac{dW^{e*}}{d\epsilon} = \frac{\epsilon}{1+n^*}\left(1 - \frac{1}{(1+n^*)^{1/3}}\right) > 0. \tag{A.26}$$

A.2.2 Quadratic Cost Function

If costs are quadratic, profits per firm are given by $\pi_i = (1-Q)q_i - 0.5c_iq_i^2 - k$, with $i \in h, l$. Ex-ante, expected profits read

$$\pi^e = 0.5(\pi_l + \pi_h) = 0.5\left((1-Q)(q_l+q_h) - 0.5c_lq_l^2 - 0.5c_hq_h^2\right) - k. \tag{A.27}$$

Market equilibrium is derived as described in the base model. Using the best-response functions and the free-entry condition, we obtain

$$n^*(\epsilon) = \frac{\left((1+0.5c)(1+c)^2 - 0.5c\epsilon^2\right)^{0.5} k^{-0.5} + \epsilon^2 - (1+c)^2}{1+c}, \tag{A.28}$$

$$\tilde{q}^*(\epsilon) = \frac{2(1+c)k^{0.5}}{\left((1+0.5c)(1+c)^2 - 0.5c\epsilon^2\right)^{0.5}}, \tag{A.29}$$

$$W^*(\epsilon) = 0.5[Q^*(\epsilon)]^2 = 0.5\left(1 - \left(\frac{k((1+c)^2-\epsilon^2)^2}{(1+0.5c)(1+c)^2 - 0.5c\epsilon^2}\right)^{0.5}\right)^2. \tag{A.30}$$

Differentiating n^* with respect to ϵ, we find

$$\frac{dn^*}{d\epsilon} = \frac{1}{1+c}\left(-0.5\left((1+0.5c)(1+c)^2 - 0.5c\epsilon^2\right)^{-0.5} c\epsilon k^{-0.5} + 2\epsilon\right), \tag{A.31}$$

which is positive if

$$k > \frac{c}{4\left((1+n^*(\epsilon)+c)(1+c) - \epsilon^2\right)} \equiv k^{crit*}(\epsilon). \tag{A.32}$$

Moreover, we obtain

$$\frac{d\tilde{q}^*}{d\epsilon} = \frac{0.5c\epsilon\tilde{q}^*}{(1+0.5c)(1+c)^2 - 0.5c\epsilon^2} > 0, \tag{A.33}$$

$$\frac{dQ^*}{d\epsilon} = 2\epsilon k^{0.5}\frac{(1+0.25c)(1+c)^2 - 0.25c\epsilon^2}{\left((1+0.5c)(1+c)^2 - 0.5c\epsilon^2\right)^{1.5}} > 0, \tag{A.34}$$

$$\frac{dW^*}{d\epsilon} = Q^*\frac{dQ^*}{d\epsilon} > 0. \tag{A.35}$$

The social planner maximizes $W^e(n) = n\pi^e(n,\epsilon) + 0.5[Q(n,\epsilon)]^2$ subject to

$$\pi^e(\epsilon, n) = \frac{(1+0.5c)(1+c)^2 - 0.5c\epsilon^2}{\left((1+n+c)(1+c) - \epsilon^2\right)^2} - k, \tag{A.36}$$

$$Q(\epsilon, n) = \frac{n(1+c)}{(1+n+c)(1+c) - \epsilon^2}. \tag{A.37}$$

This results in

$$\begin{aligned}\frac{dW^e}{dn} &= \pi^e(n) + \underbrace{n\underbrace{\frac{d\pi^e}{dn}}_{<0} + Q(n)\underbrace{\frac{dQ}{dn}}_{>0}}_{<0} \\ &= \frac{1}{\left((1+n+c)(1+c) - \epsilon^2\right)^2} M(\epsilon, n) = 0,\end{aligned} \tag{A.38}$$

with

$$M(\epsilon, n) = (1+0.5c)(1+c)^2 - 0.5c\epsilon^2 - \left((1+n+c)(1+c)-\epsilon^2\right)^2 k \\ - \frac{n(1+c)\left((1+c)^2+\epsilon^2\right)}{(1+n+c)(1+c)-\epsilon^2} = 0. \tag{A.39}$$

Eq. (A.39) implicitly determines the number of firms $n^{opt}(\epsilon)$. The socially optimal levels of expected output per firm, expected profits and expected welfare are given by

$$\tilde{q}^{opt}(\epsilon) = \frac{1+c}{(1+n^{opt}(\epsilon)+c)(1+c)-\epsilon^2}, \tag{A.40}$$

$$\pi^{eopt}(\epsilon) = \frac{(1+0.5c)(1+c)^2 - 0.5c\epsilon^2}{\left((1+n^{opt}(\epsilon)+c)(1+c)-\epsilon^2\right)^2} - k, \tag{A.41}$$

$$W^{eopt}(\epsilon) = n^{opt}(\epsilon)\pi^{eopt}(\epsilon) + 0.5Q^{opt}(\epsilon)^2. \tag{A.42}$$

Totally differentiating (A.39) yields

$$\frac{dn^{opt}}{d\epsilon} = -\frac{\partial M/\partial\epsilon}{\partial M/\partial n}, \tag{A.43}$$

$$\frac{\partial M}{\partial \epsilon} = \epsilon\left(4\left((1+n+c)(1+c)-\epsilon^2\right)k - c - \frac{2n^2(1+c)^2+4n(1+c)^3}{\left((1+n+c)(1+c)-\epsilon^2\right)^2}\right). \tag{A.44}$$

Due to the second-order condition, we have $\partial M/\partial n < 0$. Hence $dn^{opt}/d\epsilon > 0$ if

$$k > k^{critopt}(\epsilon) \equiv \frac{0.25c}{\left((1+n^{opt}(\epsilon)+c)(1+c)-\epsilon^2\right)} + \frac{n^{opt}(\epsilon)^2(1+c)^2+4n^{opt}(\epsilon)(1+c)^3}{4\left((1+n^{opt}(\epsilon)+c)(1+c)-\epsilon^2\right)^3}. \tag{A.45}$$

Comparing the critical values $k^{crit*}(\epsilon)$ and $k^{critopt}(\epsilon)$, we find that $k^{critopt} > k^{crit*}$ because $n^* > n^{opt}$ such that the first term in (A.45) already exceeds k^{crit*}. Moreover, we obtain

$$\frac{d\tilde{q}^{opt}}{d\epsilon} = \underbrace{\frac{\partial\tilde{q}^{opt}}{\partial\epsilon}}_{>0} + \underbrace{\frac{\partial\tilde{q}^{opt}}{\partial n^{opt}}}_{<0}\frac{dn^{opt}}{d\epsilon}, \tag{A.46}$$

$$\frac{dQ^{opt}}{d\epsilon} = \frac{\partial Q^{opt}}{\partial\epsilon} + \underbrace{\frac{\partial Q^{opt}}{\partial n^{opt}}}_{>0}\frac{dn^{opt}}{d\epsilon} \qquad \text{with} \tag{A.47}$$

$$\frac{\partial Q^{opt}}{\partial\epsilon} = \frac{2\epsilon n^{opt}(1+c)}{\left((1+n^{opt}+c)(1+c)-\epsilon^2\right)^2} > 0. \tag{A.48}$$

For $k \leq k^{critopt}(\epsilon)$, we have $dn^{opt}/d\epsilon \leq 0$ and $d\tilde{q}^{opt}/d\epsilon > 0$, while $dQ^{opt}/d\epsilon$ cannot be signed. For $k \geq k^{critopt}(\epsilon)$, we have $dn^{opt}/d\epsilon \geq 0$ and, therefore, $dQ^{opt}/d\epsilon > 0$. Because $(4+c)(1+c)^2 - c\epsilon^2 > 0$ holds the effect on expected welfare is positive

$$\frac{dW^{eopt}}{d\epsilon} = \epsilon n^{opt} \frac{(4+c)(1+c)^2 - c\epsilon^2 + (2+c)(1+c)n^{opt}}{((1+n^{opt}+c)(1+c)-\epsilon^2)^3} > 0. \tag{A.49}$$

Comparing the outcomes in market equilibrium with their counterparts in the social optimum, we find that the market outcome is inefficient (excessive entry and insufficient output per firm) irrespective of the value of ϵ. This can be proven by rearranging (A.39).

$$M(n^{opt}, \epsilon) = \pi^e(n^{opt}) \left((1+n^{opt}+c)(1+c)-\epsilon^2\right)^2 - \frac{n^{opt}(1+c)\left((1+c)^2+\epsilon^2\right)}{(1+n^{opt}+c)(1+c)-\epsilon^2} = 0 \tag{A.50}$$

Evaluating (A.50) at n^* and noting that $\pi^e(n^*) = 0$, we find that $M(n^*, \epsilon) < 0$. This implies $n^* > n^{opt}\ \forall \epsilon$. Given $d\tilde{q}/dn < 0$, we then obtain $\tilde{q}^* < \tilde{q}^{opt}\ \forall \epsilon$.

When analyzing the robustness of Proposition 4, we have to rely on a numerical solution of our model. Using the same parameter values as in Section 5.2, we find

	$\epsilon = 0$	$\epsilon = 0.17$
$n^*(\epsilon) - n^{opt}(\epsilon)$	1.36302	1.40455
$\tilde{q}^{opt}(\epsilon) - \tilde{q}^*(\epsilon)$	0.109065	0.114258
$Q^*(\epsilon) - Q^{opt}(\epsilon)$	0.130878	0.134358
$W^{eopt}(\epsilon) - W^*(\epsilon)$	0.0331707	0.0354954

A.2.3 Allowing for Firm Closures

In the setting with firm closures, the timing is the following. First, firms or the social planner decide about entry. Entry costs, k, are sunk. Second, each firm learns about its marginal costs. Third, firms can exit at zero costs. Finally, the remaining firms simultaneously choose output. If $k < 2\epsilon^2$ holds, all high-cost firms will surely exit the market after costs have become public information. Therefore, expected profits of firm j prior to the cost realization are

$$\pi_j^e = 0.5(\pi_{jl} + \pi_{jh}) = 0.5 \times (1 - Q - c + \epsilon)q_{jl} + 0.5 \times 0 - k. \tag{A.51}$$

The first-order condition for a profit maximum of a low-cost firm is

$$\frac{d\pi_{jl}}{dq_{jl}} = 1 - Q - c + \epsilon - q_{jl} = 0. \tag{A.52}$$

Imposing symmetry, $Q = 0.5nq_l$, output per firm, q_l, and aggregate output, Q, are

$$Q(\epsilon, n) = 0.5nq_l(\epsilon, n) = \frac{n(1 - c + \epsilon)}{2 + n}. \tag{A.53}$$

Substituting (A.53) into (A.51), expected profits are given by

$$\pi^e(\epsilon, n) = \frac{2(1 - c + \epsilon)^2}{(2 + n)^2} - k. \tag{A.54}$$

Setting expected profits equal to zero, the number of entrants in market equilibrium is

$$n^*(\epsilon) = \frac{1 - c + \epsilon}{\sqrt{0.5k}} - 2. \tag{A.55}$$

Using this information, output levels and consumer surplus are:

$$\begin{aligned} Q^*(\epsilon) &= 1 - c + \epsilon - q_l^* = 1 - c + \epsilon - \sqrt{2k}, \\ CS^*(\epsilon, n) &= \frac{n^2(1 - c + \epsilon)^2}{2(2 + n)^2} = \frac{(1 - c + \epsilon - \sqrt{2k})^2}{2}. \end{aligned} \tag{A.56}$$

In order to ascertain whether the outcome represents a stable equilibrium, suppose that one high-cost firm considers to stay in the market. It's operating profits will be (weakly) positive if $1 - c - \epsilon - Q^*(\epsilon) \geq 0$ holds. Substituting for Q^*, this condition will not be fulfilled and entry unprofitable if $\epsilon > \sqrt{0.5k}$ holds. (A.55) and (A.56) establish the validity of Proposition 2 for the present set-up, with the exception of the prediction about output per firm.

Using (A.54) and (A.56), expected welfare can be expressed as

$$W^e(\epsilon, n) = CS(\epsilon, n) + n\pi^e(\epsilon, n) = \frac{(4n + n^2)(1 - c + \epsilon)^2}{2(2 + n)^2} - nk. \tag{A.57}$$

Maximizing $W^e(\epsilon, n)$, we can compute the optimal number of firms as

$$n^{opt}(\epsilon) = \left(\frac{2(1-c+\epsilon)^2}{0.5k}\right)^{1/3} - 2 = 2\left[(0.5n^*(\epsilon)+1)^{2/3} - 1\right]. \tag{A.58}$$

To compare n^{opt} and n^*, note that any increase in n^*, for $n^* \geq 1$, results in a rise in the optimal number of firms by less than one.

$$\frac{dn^{opt}}{dn^*} = \frac{2^{4/3}}{3(n^*+2)^{1/3}} < \frac{2^{4/3}}{3(1+2)^{1/3}} < 1. \tag{A.59}$$

Suppose that n^* is minimal, i.e. $n^* = 1$. This implies $n^{opt} \approx 0.62$. Therefore, n^* is greater than n^{opt} at the lowest feasible equilibrium number of firms. This difference widens with n^*. Hence, we have excessive entry (see Proposition 1). Since the equilibrium number of firms $n^*(\epsilon)$ rises with ϵ (see (A.55)), uncertainty aggravates excessive entry.

Using (A.58), we can calculate output per firm, aggregate output, consumer surplus, expected profits and expected welfare in the socially optimal situation as

$$\begin{aligned}
&Q^{opt}(\epsilon) = 1-c+\epsilon - q_l^{opt}(\epsilon) = 1-c+\epsilon-(1-c+\epsilon)^{1/3}(2k)^{1/3},\\
&CS^{opt}(\epsilon) = 0.5(1-c+\epsilon)^2 - (1-c+\epsilon)^{4/3}(2k)^{1/3} + 0.5(1-c+\epsilon)^{2/3}(2k)^{2/3},\\
&\pi^{eopt}(\epsilon) = 0.5^{1/3}(1-c+\epsilon)^{2/3}k^{2/3} - k = 0.5kn^{opt}(\epsilon),\\
&W^{eopt}(\epsilon) = 0.5(1-c+\epsilon)^2 - 1.5(1-c+\epsilon)^{2/3}(2k)^{2/3} + 2k.
\end{aligned} \tag{A.60}$$

(A.60) shows that output per firm rises with uncertainty. Moreover, aggregate output, $Q^{opt}(\epsilon) = n^{opt}(\epsilon)q_l^{opt}(\epsilon)$, increases in the number of firms, $n^{opt}(\epsilon)$, which, in turn, rises with ϵ. Therefore, aggregate output and consumer surplus in the social optimum rise with uncertainty. Since profits and consumer surplus grow, welfare also increases with uncertainty. The above establishes the claim about Proposition 3 in the main text.

We know that ${q_l}^{opt}(\epsilon) > {q_l}^*(\epsilon)$ and that ${q_l}^{opt}(\epsilon)$ rises with ϵ. Moreover, the difference in aggregate output equals $Q^*(\epsilon) - Q^{opt}(\epsilon) = {q_l}^{opt}(\epsilon) - {q_l}^*(\epsilon)$. Hence, the positive difference between aggregate output in market equilibrium and socially optimal output rises. Finally, the difference in welfare levels is given by

$$\begin{aligned} W^{eopt}(\epsilon) - W^*(\epsilon) &= 0.5(1-c+\epsilon)^2 - 1.5(1-c+\epsilon)^{2/3}(2k)^{2/3} + 2k - 0.5(1-c+\epsilon-\sqrt{2k})^2 \\ &= k + (1-c+\epsilon)\sqrt{2k} - 1.5(1-c+\epsilon)^{2/3}(2k)^{2/3}. \end{aligned} \tag{A.61}$$

The derivative with respect to ϵ,

$$\frac{d(W^{eopt} - W^*)}{d\epsilon} = \sqrt{2k}\left(1 - \frac{(2k)^{1/6}}{(1-c+\epsilon)^{1/3}}\right), \tag{A.62}$$

can be rewritten by substituting

$$1 + 0.5n^{opt}(\epsilon) = \frac{(1-c+\epsilon)^{2/3}}{(2k)^{1/3}} \tag{A.63}$$

from (A.58), yielding

$$\frac{d(W^{eopt} - W^*)}{d\epsilon} = \sqrt{2k}\left(1 - \frac{1}{\sqrt{0.5n^{opt}(\epsilon)+1}}\right) > 0. \tag{A.64}$$

These computations illustrate the statement concerning Proposition 4 in the main text.

A.2.4 Integer Constraint and Uncorrelated Probabilities

Assume that the number of firms is an integer and the probability of having low marginal cost for each firm is 0.5 ex-ante and ex-post. This probability is independent of the probability that other firms have low or high marginal costs. In consequence, aggregate output is uncertain ex-ante, and there is comprehensive uncertainty. The timing is the same as in the model outlined in Section 3.

Aggregating the first-order condition for the optimal choice of output (see (4)) and using $Q = \sum_{j=1}^{n} q_j$ and $C = \sum_{j=1}^{n} c_j$, we obtain

$$n - nQ - Q - C = 0 \Rightarrow Q = \frac{n - C}{1 + n}. \tag{A.65}$$

Suppose, that firm j competes with m high-cost firms. If firm j's production costs are also high, there are $m+1$ high-cost firms, such that total costs are

$$C(n, m+1, \epsilon) = (m+1)(c+\epsilon) + (n-(m+1))(c-\epsilon) = nc + \epsilon(2(m+1)-n). \quad (A.66)$$

Using (4) and (A.66), output of a high-cost firm j, can be written as

$$q_{jh}(n, m+1, \epsilon) = \frac{1 - c_h(1+n) + C(n, m+1, \epsilon)}{1+n} = \frac{1 - c + \epsilon(2m+1-2n)}{1+n}. \quad (A.67)$$

If firm j is a low-cost firm, there are m high-cost firms, and total costs, C, are

$$C(n, m, \epsilon) = nc + \epsilon(2m - n). \quad (A.68)$$

Combining (4) and (A.68), output of a low-cost firm j is

$$q_{jl}(n, m, \epsilon) = \frac{1 - c_l(1+n) + C(n, m, \epsilon)}{1+n} = \frac{1 - c + \epsilon(1+2m)}{1+n}. \quad (A.69)$$

To compute expected profits, we use the feature that profits equal $\pi_{ji} = (q_{ji})^2 - k$ (from (1) and (4)). If there are m other high costs firms in the market, there will be $m+1$ high-cost and $n-m-1$ low-costs producers if the firm under consideration is also a high-cost firm. The number of high-cost firms, respectively low-cost firms, drops, respectively rises, by one if the firm's marginal costs are low. The ex-post number of high-cost firms, m, can vary between zero and n, since cost realizations are uncorrelated. The probability function for the binomial random variable is given by $b(m, n, 0.5) = \binom{n}{m}(0.5)^m(0.5)^{n-m} = \binom{n}{m}(0.5)^n$. Therefore, expected profits of this firm, which does not yet know whether its own production costs will be high or low, are given by

$$\begin{aligned}
\pi^{e,int}(n, \epsilon) &= \sum_{m=0}^{n-1} \left(\frac{(n-1)!}{m!(n-1-m)!} \right) \left(\frac{1}{2} \right)^{n-1} \left(\frac{1 - c + \epsilon(1+2m-2n)}{1+n} \right)^2 \\
&+ \sum_{m=0}^{n-1} \left(\frac{(n-1)!}{m!(n-1-m)!} \right) \left(\frac{1}{2} \right)^{n-1} \left(\frac{1 - c + \epsilon(1+2m)}{1+n} \right)^2 - k \\
&= \frac{(1-c)^2 + \epsilon^2(n+n^2-1)}{(1+n)^2} - k.
\end{aligned} \quad (A.70)$$

The expression in (A.70) declines in n if ϵ is not too large. Given this constraint, the (integer) number of firms in market equilibrium is given by $n^{*,int}(\epsilon)$, such that $\pi^{e,int}(\epsilon, n^{*,int}) > 0 > \pi^{e,int}(\epsilon, n^{*,int}+1)$. Expected profits for $c = 0.2$, $k = 0.06$ and $\epsilon = 0$ are positive for $n = 2$ ($\pi^{e,int}(0,2) = 0.01$) and negative for $n = 3$ ($\pi^{e,int}(0,3) = -0.02$). Hence, two firms enter the market in equilibrium. The respective numbers for other values of ϵ and k, as depicted in Table 4, are computed in an analogous manner.

If there are m high-cost firms and $n - m$ low-cost ones, expected aggregate output is

$$Q^{e,int}(n,m,\epsilon) = mq_h(n,m,\epsilon) + (n-m)q_l(n,m,\epsilon) = \frac{n(1-c) + \epsilon(n-2m)}{1+n}. \tag{A.71}$$

Expected consumer surplus can be expressed as

$$CS^{e,int}(n,\epsilon) = \sum_{m=0}^{n} \frac{n!}{m!(n-m)!} \frac{(Q^{e,int}(n,m,\epsilon))^2}{2} = \frac{n^2(1-c)^2 + n\epsilon^2}{2(1+n)^2}. \tag{A.72}$$

Expected welfare is the sum of aggregate expected profits and expected consumer surplus

$$\begin{aligned} W^{e,int}(n,\epsilon) &= n \times \pi^{e,int}(n,\epsilon) + CS^{e,int}(n,\epsilon) \\ &= n\frac{(1-c)^2(2+n) + \epsilon^2(2n + 2n^2 - 1)}{2(1+n)^2} - nk. \end{aligned} \tag{A.73}$$

Substituting $c = 0.2$, $k = 0.06$, and $\epsilon = 0$, we obtain $W^{e,int}(1,0) = 0.18$ and $W^{e,int}(2,0) = 0.164$. Hence, the socially optimal number of firms is one in the absence of uncertainty. The other values depicted in Table 4 are calculated in an analogous manner.

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